Algebra III

Curriculum Framework

2012

Course Title: Algebra III
Course/Unit Credit: 1
Course Number:
Teacher Licensure: Secondary Mathematics
Grades: 9-12

Algebra III

This course will enhance the higher level thinking skills developed in Algebra II through a more in-depth study of those concepts and exploration of some pre-calculus concepts. Students in Algebra III will be challenged to increase understanding of algebraic, graphical, and numerical methods to analyze, translate and solve polynomial, rational, exponential, and logarithmic functions. Modeling real world situations is an important part of this course. Sequences and series will be used to represent and analyze real world problems and mathematical situations. Algebra III will also include a study of matrices and conics. Arkansas teachers are responsible for integrating appropriate technology and including the eight Standards for Mathematical Practice found in the Common Core State Standards for Mathematics (CCSS-M). Algebra III does not require Arkansas Department of Education approval.

Prerequisites: Algebra I, Geometry, Algebra II

Strand	Content Standard
Matrix Operations	
	1. Students will perform operations with matrices and use them to solve systems of equations.
Conic Sections	
	2. Students will identify, analyze, and sketch the graphs of the conic sections and relate the equations and graphs.
Function Operations and Properties	
	3. Students will be able to find the inverse of functions and use composition of functions to prove that two functions are inverses.
Interpreting Functions	
	4. Students will be able to interpret different types of functions and key characteristics including polynomial, exponential, logarithmic, and rational functions.
Sequences and Series	
	5. Students will use sequences and series to represent and analyze mathematical situations.

Strand: Matrix Operations

Content Standard 1: Students will perform operations with matrices and use them to solve systems of equations.

		Connections to CCSS-M
MO.1.AIII.1	Use matrices to represent and manipulate data (e.g., to represent payoffs or incidence relationships in a network)	N.VM.6
MO.1.AIII.2	Multiply matrices by scalars to produce new matrices (e.g., as when all of the payoffs in a game are doubled)	N.VM.7
MO.1.AIII.3	Add, subtract, and multiply matrices of appropriate dimensions	N.VM.8
MO.1.AIII.4	Understand that, unlike multiplication of numbers, *matrix multiplication* for square matrices is not a commutative operation, but still satisfies the associative and distributive properties	N.VM.9
MO.1.AIII.5	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers; the *determinant* of a square matrix is nonzero if and only if the matrix has a multiplicative inverse	N.VM.10
MO.1.AIII.6	Multiply a *vector* (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another *vector*; work with matrices as *transformations* of *vectors*	N.VM.11
MO.1.AIII.7	Work with $2 X 2$ matrices as *transformations* of the plane; interpret the absolute value of the *determinant* in terms of area	N.VM.12
MO.1. AIII.8	Represent a system of linear equations as a single matrix equation in a *vector variable*	A.REI.8
MO.1. AIII.9	Find the inverse of a matrix if it exists; use the inverse to solve systems of linear equations using technology for matrices of dimension $3 X 3$ or greater	A.REI.9

Key: MO.1.AIII.1 – Matrix Operations.Content Standard 1.Algebra III.1st Student Learning Expectation

Strand: Conic Sections

Content Standard 2: Students will identify, analyze, and sketch the graphs of the conic sections and relate the equations and graphs.

		Connections to CCSS-M
CS.2. AIII.1	Find the *conjugate* of a *complex number*; use *conjugate*s to find *moduli* and quotients of *complex numbers*	N.CN.3
CS.2. AIII.2	Derive the equations of ellipses and hyperbolas given the *foci*, using the fact that the sum or difference of distances from the *foci* is constant; find the equations for the *asymptotes* of a hyperbola	G.GPE.3
CS.2. AIII.3	Complete the square in order to generate an equivalent form of an equation for a *conic section;* use that equivalent form to identify key characteristics of the *conic section*	Not Applicable
CS.2. AIII.4	Identify, graph, write, and analyze equations of each type of *conic section*, using properties such as *symmetry*, intercepts, *foci*, *asymptotes*, and *eccentricity*, and using technology when appropriate	Not Applicable
CS.2. AIII.5	Solve systems of equations and inequalities involving conics and other types of equations, with and without appropriate technology	Not Applicable

Key: CS.2.AIII.1 – Conic Sections.Content Standard 2.Algebra III.1st Student Learning Expectation

Strand: Function Operations and Properties

Content Standard 3: Students will be able to find the inverse of functions and use composition of functions to prove that two functions are inverses.

		Connections to CCSS-M
FOP.3.AIII.1	Compose functions (e.g., if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time)	F.BF.1c
FOP.3. AIII.2	Verify, by *composition,* that one function is the inverse of another	F.BF.4b
FOP.3. AIII.3	Read values of an *inverse function* from a graph or a table, given that the function has an inverse	F.BF.4c
FOP.3. AIII.4	Produce an *invertible function* from a *non-invertible function* by restricting the domain	F.BF.4d
FOP.3. AIII.5	Combine standard function types using arithmetic operations (e.g., build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential and relate these functions to the model)	F.BF.1b
FOP.3. AIII.6	Understand the inverse relationship between exponents and *logarithms;* use this relationship to solve problems involving *logarithms* and exponents	F.BF.5
FOP.3.AIII.7	Graph *transformations* of functions including quadratic, absolute value, square root, cube root, cubic, and step functions; graph *piece-wise* defined functions including these *transformations*	G.CO.2
FOP.3. AIII.8	Determine numerically, graphically, and algebraically if a function is even, odd, or neither	F.IF.9

Key: FOP.3.AIII.1 – Function Operations and Properties.Content Standard 3.Algebra III.1st Student Learning Expectation

Strand: Interpreting Functions

Content Standard 4: Students will be able to interpret different types of functions and key characteristics including polynomial, exponential, logarithmic, and rational functions.

		Connections to CCSS-M
IF.4. AIII.1	Graph rational functions identifying zeros and *asymptotes* when suitable factorizations are available; show end behavior	F.IF.7d
IF.4. AIII.2	Analyze and interpret polynomial functions numerically, graphically, and algebraically, identifying key characteristics such as intercepts, end behavior, domain and range, relative and absolute *maximum* and *minimum*, as well as intervals over which the function increases and decreases	F.IF.4
IF.4. AIII.3	Analyze and interpret rational functions numerically, graphically, and algebraically, identifying key characteristics such as *asymptotes* (*vertical, horizontal*, and slant), end behavior, point discontinuities, intercepts, and domain and range	F.IF.7d
IF.4. AIII.4	Analyze and interpret *exponential functions* numerically, graphically, and algebraically, identifying key characteristics such as *asymptotes*, end behavior, intercepts, and domain and range	F.IF.7e
IF.4. AIII.5	Analyze and interpret *logarithmic functions* numerically, graphically, and algebraically, identifying key characteristics such as *asymptotes*, end behavior, intercepts, and domain and range	F.IF.7e
IF.4. AIII.6	Build functions to model real-world applications using algebraic operations on functions and *composition of functions,* with and without appropriate technology [e.g., profit functions as well as volume and surface area (*optimization* subject to constraints)]	F.BF.1d, F.BF.1c

Key: IF.4.AIII.1 – Interpreting Functions.Content Standard 4.Algebra III.1st Student Learning Expectation

Strand: Sequences and Series

Content Standard 5: Students will use sequences and series to represent and analyze mathematical situations.

		Connections to CCSS-M
SS.5. AIII.1	Write *arithmetic* and *geometric sequences* both recursively and with an explicit formula; translate between the two forms	F.BF.2
SS.5. AIII.2	Use *arithmetic* and *geometric sequences* both recursively and with an explicit formula to model situations	F.BF.2
SS.5. AIII.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers	F.IF.3
SS.5. AIII.4	Use sequences and series to solve real world problems, with and without appropriate technology	Not Applicable

Key: SS.5.AIII.1 – Sequences and Series.Content Standard 5.Algebra III.1st Student Learning Expectation

Arithmetic Sequence	A sequence in which each term after the first is found by adding a constant, called the common difference $d,$ to the previous term
Asymptote(s)	Line(s) to which a graph becomes arbitrarily close as the value of x or y increases or decreases without bound (vertical, horizontal, slant)
Complex number(s)	Number(s) that can be written as the sum or difference of a real number and an imaginary number (e.g., $5 + 2i$)
Composition of Functions	Suppose *f* and *g* are functions such that the range of *g* is a subset of the domain of *f*, then the composite function *f* of *g* can be described by the equation [$[f \circ g](x) = f[g(x)]$]
Conic section	Any figure that can be formed by slicing a double cone with a plane
Conjugate(s)	The result of writing the sum of two terms as a difference or vice-versa
Determinant	A single number obtained from a matrix that reveals a variety of the matrix's properties
Eccentricity	A number that indicates how drawn out or attenuated a conic section is; eccentricity is represented by the letter e (no relation to $\mathrm{e} = 2.718...$)
Exponential Function	A function in which the variable(s) occurs in the exponent [e.g., $f(x) = ab^{x}, b > 0$]
Foci	Two fixed points on the interior of an ellipse used in the formal definition of the curve
Geometric Sequence	A sequence in which each term after the first is found by multiplying the previous term by a constant, called the common ratio, r
Horizontal Asymptote	A horizontal line to which a graph becomes arbitrarily close as the value of x increases or decreases without bound
Inverse Function	Two functions f and g are inverse functions, if and only if, both of their *composition*s yield the identity function; p for example, $[f \circ g](x) = x$ and $[g \circ f](x) = x$
Invertible function	A function that has an inverse
Logarithmic Function	A function of the form $y = log_{b}x$, where $b > 0, x > 0$ and $b \neq 1$
Logarithm(s)	The logarithm base *b* of a number x is the power to which b must be raised in order to equal x; this is written $log_{b}\,\mathrm{x}$ (e.g., $log_{3}\,81$ *equals* 4 *since* $3^{4} = 81$)

Matrix multiplication	Given Matrix *A* with dimensions $[g\ X\ h]$ and Matrix *B* with dimensions $[y\ X\ z]$; the matrix multiplication of *AB* results in a matrix with dimensions of $[g\ X\ z]$ and is only possible if the number of columns in *A* is equal to the number of rows in *B* (if and only if $h\ =\ y$); matrices are multiplied as shown below: $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f & g \\ h & i & j \end{bmatrix} = \begin{bmatrix} ae + bh & af + bi & ag + bj \\ ce + dh & cf + di & cg + dj \end{bmatrix}$ Example: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 5 & 6 & 7 \\ 8 & 9 & 10 \end{bmatrix} = \begin{bmatrix} 1 \cdot 5 + 2 \cdot 8 & 1 \cdot 6 + 2 \cdot 9 & 1 \cdot 7 + 2 \cdot 10 \\ 3 \cdot 5 + 4 \cdot 8 & 3 \cdot 6 + 4 \cdot 9 & 3 \cdot 7 + 4 \cdot 10 \end{bmatrix}$
Maximum	The greatest value of a function if the function has such an extreme value
Minimum	The least value of a function if the function has such an extreme value
Moduli	Distance of the complex number from the origin in a complex plane
Non-invertible function	A function that does not have an inverse
Optimization	The process by which one seeks to minimize or maximize a real function by systematically choosing the values of real or integer variables from within a domain
Piece-wise Function	A function defined by different rules for different parts of the domain
Symmetry	A figure has symmetry if the figure and its image coincide after a transformation
Transformations of graphs	Operations that alter the form of a figure (e.g., horizontal shifts, vertical shifts, horizontal stretches, vertical stretches, reflections)
Transformations of vectors	For each point $P(x, y)$ in the plane there is a corresponding position vector $\begin{bmatrix} x \\ y \end{bmatrix}$ from the origin $(0,0)$ to the point (x, y). The motion of the point (x, y) under a geometric transformation (for example, a rotation or reflection) can be modeled as the multiplication of a 2 by 2 transformation matrix times the position vector $\begin{bmatrix} x \\ y \end{bmatrix}$
Vector	A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers
Vector (variable)	A vector whose components are variable. For example, if a system of linear equations is written in matrix form $Ax\ =\ b$, where A is the coefficient matrix and b is the vector of the constants, $x = \begin{bmatrix} x \\ y \end{bmatrix}$ is an unknown vector variable
Vertical Asymptote	A vertical line to which a graph becomes arbitrarily close as the value of $f(x)$ increases or decreases without bound