Pre-Cal including Trigonometry

Mathematics
Curriculum Framework

Revised 2004

Course Title: Pre-Calculus Including Trigonometry (Fourth-year Course)
Course/Unit Credit: 1
Course Number:
Teacher Licensure: Secondary Mathematics
Pre-requisite: Algebra II
Grades: 9-12

Pre-Calculus including Trigonometry

Pre-Calculus including trigonometry is designed for students who have successfully completed Algebra II and Geometry. Students will use symbolic reasoning and analytical methods to represent mathematical situations, to express generalizations, and to study mathematical concepts and the relationships among them. Students will use functions and equations as tools for expressing generalizations. This course will emphasize a study of trigonometric functions and identities as well as applications of right triangle trigonometry and circular functions. Students will be introduced to polar coordinates in this class. Arkansas teachers will be responsible for integrating appropriate technology in the Pre-Calculus curriculum.

Strand	Standard
Polynomial and Rational Functions	
	1. Students will analyze polynomial and rational functions graphically and algebraically.
Exponential and Logarithmic Functions	
	2. Students will solve real world problems involving logarithmic and exponential functions. Draw and analyze graphs and find inverse functions.
Conics	
	3. Students will identify, analyze and sketch the graphs of the conic sections and relate their equations and graphs.
Sequences and Series	
	4. Students will use sequences and series to represent, analyze, and solve real world problems and mathematical situations.
Trigonometric Functions	
	5. Students will use different perspectives to develop and apply the definitions of the six trigonometric functions. They will sketch and analyze graphs, find inverse functions, and solve real world problems.
Oblique Triangles	
	6. Students will identify, create, and solve real world problems involving oblique triangles and vectors.
Trigonometric Equations and Identities	
	7. Students will verify trigonometric identities and solve trigonometric equations.
Polar Coordinates	
	8. Students will define polar coordinates and relate them to rectangular coordinates.

Polynomial and Rational Functions

CONTENT STANDARD 1. Students will analyze polynomial and rational functions graphically and algebraically.

PRF.1.PCT.1	Investigate and sketch, with and without appropriate technology, the graphs of *polynomia*l and *rational functions* using the characteristics of domain and range, upper and *lower bounds*, *maximum* and *minimum* points, *asymptotes* and *end behavior*, *zeros*, *multiplicity of zeros*, y-intercepts, and *symmetry*
PRF.1.PCT.2	Solve, with and without appropriate technology, polynomial equations utilizing techniques such as Descartes' Rule of Signs, upper and lower bounds, Intermediate Value Theorem and Rational Root Theorem
PRF.1.PCT.3	Describe, with and without appropriate technology, the fundamental characteristics of rational functions: zeros, *discontinuities* (including *vertical asymptotes*), and end behavior (including *horizontal asymptotes*)
PRF.1.PCT.4	Apply the concepts of polynomial and rational functions to model real world situations using appropriate technology when needed

Key: PRF.1.PCT.1 = Polynomial and Rational Functions. Standard 1. Pre-Calculus including Trigonometry. 1st Student Learning Expectation

Exponential and Logarithmic Functions

CONTENT STANDARD 2. Students will solve real world problems involving logarithmic and exponential functions. Draw and analyze graphs and find inverse functions.

ELF.2.PCT.1	Establish the inverse relationship between *exponential* and *logarithmic functions*
ELF.2.PCT.2	Develop and apply the laws of logarithms and the change-of-base formula to simplify and evaluate expressions
ELF.2.PCT.3	Solve graphically, algebraically and numerically, with and without appropriate technology, equations and real world problems involving exponential and logarithmic expressions
ELF.2.PCT.4	Find, with and without appropriate technology, the domain, range, intercepts, and asymptotes of logarithmic and exponential functions
ELF.2.PCT.5	Draw and analyze, with and without appropriate technology, graphs of logarithmic and exponential functions

Key: ELF.2.PCT.1 = Exponential and Logarithmic Functions. Standard 2. Pre-Calculus including Trigonometry. 1st Student Learning Expectation

Conics

CONTENT STANDARD 3. Students will identify, analyze and sketch the graphs of the conic sections and relate their equations and graphs.

C.3.PCT.1	Identify, graph, write, and analyze equations of *conic sections*, using properties such as symmetry, intercepts, foci, asymptotes, and *eccentricity*, and when appropriate, use technology
C.3.PCT.2	Solve, with and without appropriate technology, systems of equations and inequalities involving conics and other types of equations
C.3.PCT.3	Solve, with and without appropriate technology, real world problems involving conic sections

Key: C.3.PCT.1 = Conics. Standard 3. Pre-Calculus including Trigonometry. 1st Student Learning Expectation

Sequences and Series

CONTENT STANDARD 4. Students will use sequences and series to represent, analyze, and solve real world problems and mathematical situations.

SS.4.PCT.1	Develop, with and without appropriate technology, a representation of *sequences recursively*
SS.4.PCT.2	Define and discriminate between *arithmetic* and *geometric sequences* and *series* and use appropriate technology when needed
SS.4.PCT.3	Solve, with and without appropriate technology, problems involving the sum (including *Sigma notation*) of *finite* and *infinite* sequences and series
SS.4.PCT.4	Determine the n^{th} term of a sequence given a rule or specific terms and use appropriate technology when needed
SS.4.PCT.5	Use, with and without appropriate technology, sequences and series to solve real world problems

Key: SS.4.PCT.1 = Sequences and Series. Standard 4. Pre-Calculus including Trigonometry. 1st Student Learning Expectation

Trigonometric Functions

CONTENT STANDARD 5. Students will use different perspectives to develop and apply the definitions of the six trigonometric functions. They will sketch and analyze graphs, find inverse functions, and solve real world problems.

TF.5.PCT.1	Define the six trigonometric functions as • *circular functions* • ratios of sides of right triangles • functions of an angle in *standard position* when given a point on the terminal side of the angle
TF.5.PCT.2	Use degrees and *radians* interchangeably to represent angle measure
TF.5.PCT.3	Sketch an angle in standard position and determine the *reference angle* and *coterminal angles*
TF.5.PCT.4	Find the values of the trigonometric functions given the value of one trigonometric function and an additional piece of qualifying information or given the coordinates of a point on the terminal side of an angle
TF.5.PCT.5	Develop and become fluent in the recall of the exact values of the trigonometric functions for special angles
TF.5.PCT.6	Solve, with and without appropriate technology, real world problems involving applications of trigonometric functions
TF.5.PCT.7	Graph the six trigonometric functions, identify domain, range, intercepts, *period*, *amplitude*, and asymptotes as applicable and use symmetry to determine whether the function is *even* or *odd* through appropriate technology when needed
TF.5.PCT.8	Determine, with and without appropriate technology, the amplitude, period, *phase shift*, and vertical shift, and sketch the graph of transformations of the trigonometric functions
TF.5.PCT.9	Identify and graph, with and without appropriate technology, the inverse of trigonometric functions including the restrictions on the domain

Key: TF.5.PCT.1 = Trigonometric Functions. Standard 5. Pre-Calculus including Trigonometry. 1st Student Learning Expectation

Oblique Triangles

CONTENT STANDARD 6. Students will identify, create, and solve real world problems involving oblique triangles and vectors.

OT.6.PCT.1	Develop and use the Law of Sines and the Law of Cosines to solve *oblique triangles* and use appropriate technology when needed
OT.6.PCT.2	Solve real world problems applying the Law of Sines and the Law of Cosines and appropriate technology when needed
OT.6.PCT.3	Determine the area of an oblique triangle by using an appropriate formula and appropriate technology when needed
OT.6.PCT.4	Use *vectors* to solve problems and describe addition of vectors and multiplication of a vector by a *scalar*, both symbolically and geometrically
OT.6.PCT.5	Use vectors to model situations defined by magnitude and direction and analyze and solve real world problems by using appropriate technology when needed

Key: OT.6.PCT.1 = Oblique Triangles. Standard 6. Pre-Calculus including Trigonometry. 1st Student Learning Expectation

Trigonometric Equations and Identities

CONTENT STANDARD 7. Students will verify trigonometric identities and solve trigonometric equations.

TEI.7.PCT.1	Develop the Pythagorean Identities and use to verify other identities and simplify expressions
TEI.7.PCT.2	Develop and use trigonometric formulas including sum and difference formulas and multiple-angle formulas
TEI.7.PCT.3	Solve trigonometric equations algebraically and graphically and use appropriate technology when needed

Key: TEI.7.PCT.1 = Trigonometric Equations and Identities. Standard 7. Pre-Calculus including Trigonometry. 1st Student Learning Expectation

Polar Coordinates

CONTENT STANDARD 8. Students will define polar coordinates and relate them to rectangular coordinates.

PC.8.PCT.1	Convert *polar coordinates* to rectangular coordinates and rectangular coordinates to polar coordinates
PC.8.PCT.2	Represent equations given in rectangular coordinates in terms of polar coordinates
PC.8.PCT.3	Graph *polar equations* and use appropriate technology when needed
PC.8.PCT.4	Apply *polar coordinates* to real world situations and use appropriate technology when needed

Key: PC.8.PCT.1 = Polar Coordinates. Standard 8. Pre-Calculus including Trigonometry. 1st Student Learning Expectation

Amplitude	In the equation $y = A \sin x$ or $y = A \cos x$, the amplitude is given by the $\lvert A \rvert$.
Arithmetic Sequence	A sequence in which each term after the first is found by adding a constant, called the common difference, *d to* the previous term
Arithmetic Series	The indicated sum of the terms of an arithmetic sequence
Asymptote	A line to which a graph becomes arbitrarily close as the value of x or y increases or decreases without bound (vertical, horizontal, slant)
Circular Functions	The six basic trigonometric functions defined using a unit circle
Conic Section	Any figure that can be formed by slicing a double cone with a plane
Coterminal Angles	Two angles in standard position having the same terminal side
Discontinuity	A point in the domain of a function at which the function is not continuous
Eccentricity	For a conic, the ratio of the distance of a point from a fixed point to its distance from a fixed line
End Behavior	A reference to the graph of a polynomial function as rising or falling to the right and rising or falling to the left
Exponential Functions	A function in which variable(s) occur in exponent(s)
Finite Sequence	A finite sequence with *n* terms is a function whose domain is the set of integers {1, 2, 3, …, *n*}
Finite Series	The indicted sum of a finite sequence
Geometric Sequence	A sequence in which each term after the first is found by multiplying the previous term by a constant called the common ratio, *r*
Geometric Series	The indicated sum of the terms of a geometric series

Horizontal Asymptote	A horizontal line to which a graph becomes arbitrarily close as the value of x increases or decreases without bound.
Infinite Sequence	An infinite sequence is a function whose domain is the set of positive integers.
Infinite Series	The indicated sum of an infinite series
Logarithmic Functions	A function of the form $y = \log_b x$, where $b>0$ and $b\neq 1$
Lower Bound	A number which is less than or equal to every number in the set
Maximum	The greatest value of the function if it has such an extreme value
Minimum	The least value of the function if it has such an extreme value
Multiplicity of Zeros	The number of times that a repeated zero of a function occurs
Oblique Triangles	Triangles that have no right angles
Period	The interval of the domain over which the function repeats
Phase Shift	The horizontal shift of a trigonometric function
Polar Coordinates	The system of coordinates in which a point is located by its distance from a fixed point and the angle that the line from this point to the given point makes with a fixed line, called the polar axis
Polar Equation	An equation in polar coordinates
Polynomial Functions	A function that can be described by an equation of the form P(x) = $a_0x^n + a_1x^{n-1} + ... + a_{n-2}x^2 + a_{n-1}x + a_n$, where the coefficients $a_0, a_1, a_2, ..., a_n$ represent real numbers, a_0 is not zero, and *n* represents a nonnegative integer
Radians	A central angle subtended in a circle by an arc whose length is equal to the radius of the circle

Rational Functions	An equation of the form $f(x) = \frac{p(x)}{q(x)}$, where *p(x)* and *q(x)* are polynomial functions and $q(x) \neq 0$.
Recursive Sequence	When given one or more of the first few terms, all other terms of the sequence are then defined using previous terms.
Scalar Multiplication (Vectors)	The product of a scalar *a* and a vector v is the vector having the same direction as v and of length equal to the product of a and the length of v
Sigma Notation	Notation that uses the Σ symbol to indicate a sum of a series
Standard Position	The horizontal distance from any point on the graph of a function to that point where the graph begins to repeat
Symmetry	A figure has symmetry if the figure and its image coincide after a transformation.
Upper Bounds	A number that is greater than or equal to every number in the set
Vector	A quantity that is described by both magnitude and direction
Vertical Asymptotes	A vertical line to which a graph becomes arbitrarily close as the value of f(x) increases or decreases without bound
Zeros	For any function *f(x)*, if *f(a)* = 0, then a is a zero of the function.