Calculus

Curriculum Framework

2012

Course Title: Calculus
Course/Unit Credit: 1
Course Number:
Teacher Licensure: Secondary Mathematics
Grades: 9-12

Calculus

Calculus is a two-semester course designed to provide students with experience in the methods and applications of calculus and to develop an understanding of its concepts. This course emphasizes a multi-representational approach to Calculus, with concepts, results, and problems being expressed graphically, numerically, symbolically, analytically, and verbally through the use of unifying themes of derivatives, integrals, limits, application and modeling, and approximation. Teachers are responsible for including the eight Standards for Mathematical Practice found in the Common Core State Standards for Mathematics (CCSS-M). Calculus does not require Arkansas Department of Education approval.

Prerequisites: Students must have successfully completed coursework for Algebra I, Geometry, and either CCSS Algebra II or Pre-Calculus under the 2004, amended 2006, Arkansas Mathematics Curriculum Frameworks.

Strand	Content Standard
Limits and Continuity	
	1. Students will determine the limit of a function at a value numerically, graphically, and analytically.
Derivatives	
	2. Students will use derivatives to solve problems both theoretically and in real-world context.
Integrals	
	3. Students will apply the techniques of integration to solve problems both theoretically and in contextual models that represent real-world phenomena.

Note: There are no CCSS-M connections in this course due to the fact that all of the content is above and beyond the scope of the CCSS-M.

Strand: Limits and Continuity

Content Standard 1: Students will determine the limit of a function at a value numerically, graphically, and analytically.

LC.1.C.1	Identify vertical asymptotes in rational and logarithmic functions by identifying locations where the function value approaches infinity; estimate *limits* numerically and graphically; calculate *limits* analytically: • algebraic simplification • direct substitution • one-sided limits • rationalization
LC.1.C.2	Calculate *infinite limits* and use the result to identity vertical asymptotes in rational and logarithmic functions
LC.1.C.3	Calculate *limits* at infinity and use the result to identify horizontal asymptotes in rational and exponential functions
LC.1.C.4	Calculate *limits* at infinity and use the result to identify unbounded behavior in rational, exponential, and logarithmic functions
LC.1.C.5	Identify and classify graphically, algebraically, and numerically if a discontinuity is removable or non-removable; identify the three conditions that must exist in order for a function to be continuous at $x = a$: • $f(a)$ is defined • the *limit* as x approaches a of $f(x)$ equals $f(a)$ • the *limit* as x approaches a of $f(x)$ exists
LC.1.C.6	Apply the *Intermediate Value Theorem* for *continuous functions*

Key: LC.1.C.1 – Limits and Continuity.Content Standard 1.Calculus.1st Student Learning Expectation

Strand: Derivatives

Content Standard 2: Students will use derivatives to solve problems both theoretically and in real-world context.

D.2.C.1	Approximate the *derivative*: • graphically by finding the slope of a *tangent line* drawn to a curve at a given point • numerically by using the *difference quotient*
D.2.C.2	Find the equation of the *tangent line* using the definition of *derivative*
D.2.C.3	Establish and apply the relationship between *differentiability* and continuity
D.2.C.4	Compare the characteristic of graphs of f and f': • generate the graph of f given the graph of f' and vice versa • establish the relationship between the increasing and decreasing behavior of f and the sign of f' • identify maxima and minima as points where increasing and decreasing behavior change
D.2.C.5	Apply the *Mean Value Theorem* on a given interval
D.2.C.6	Compare the characteristic of graphs of f, f', and f'': • generate the graphs of f and f' given the graph of f'' and vice versa • establish the relationship between the *concavity* of f and the sign of f'' • identify points of inflection as points where *concavity* changes
D.2.C.7	Find *derivatives* of functions using: • *Power rule* • *Product rule* • *Quotient rule*
D.2.C.8	Find *derivatives* of: • an implicitly defined equation • composite functions using *chain rule* • exponential and logarithmic functions • functions requiring the use of more than one differentiation rule
D.2.C.9	Find the equation of: • a line tangent to the graph of a function at a point • a normal line to the graph of a function at a point
D.2.C.10	Solve application problems involving: • optimization • related rates
D.2.C.11	Interpret the *derivative* as a rate of change and varied applied contexts, including velocity, speed, and *acceleration*

Key: D.2.C.1 – Derivatives.Content Standard 2.Calculus.1st Student Learning Expectation

Strand: Integrals

Content Standard 3: Students will apply the techniques of integration to solve problems, both theoretically and in contextual models that represent real-world phenomena.

I.3.C.1	Define the definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval $\int_a^b f'(x)dx = f(b) - f(a)$
I.3.C.2	Determine the area between two curves, and identify the definite integral as the area of the region bounded by two curves
I.3.C.3	Apply the *Fundamental Theorem of Calculus* to solve contextual models that represent real-world phenomena
I.3.C.4	Compute *indefinite integrals*
I.3.C.5	Determine the antiderivative of a function using rules of basic differentiation, and solve problems using the techniques of antidifferentiation
I.3.C.6	Estimate definite integrals by using Riemann sums and trapezoidal sums, and identify the definite integral as a *limit* of Riemann sums
I.3.C.7	Explore and apply different integration techniques

Key: I.3.C.1 – Integrals.Content Standard 3.Calculus.1st Student Learning Expectation

Acceleration	The rate of change of velocity over time
Chain Rule	A method for finding the derivative of a composition of functions; the formula is $\frac{d}{dx}f\big(g(x)\big) = f'\big(g(x)\big)g'(x)$
Concavity	If a curve is concave up (convex), the graph of the curve is bent upward, like an upright bowl. If a curve is concave down (or simply concave), then the graph of the curve is bent down, like a bridge. For a function $f(x)$ where $f(x)$ and $f'(x)$ are both differentiable, $f(x)$ is concave up if $f''(x) \geq 0$ and concave down if $f''(x) \leq 0$. If $f''(x) = 0$, then *x* is an inflection point, where the graph changes direction of concavity
Continuous function(s)	A function is continuous at $x = a$ if 1. $\lim_{x\to a} f(x)$ exists 2. $f(a)$ exists 3. $\lim_{x\to a} f(x) = f(a)$
Derivative(s)	A function which gives the slope of a curve; that is, the slope of the line tangent to a function; the derivative of a function *f* at a point *x* is commonly written *f* '(*x*)
Difference quotient	For a function *f*, the formula $\frac{(x+h)-f(x)}{h}$; this formula computes the slope of the secant line through two points on the graph of *f*; these are the points with *x*-coordinates *x* and *x* + *h*; the difference quotient is used in the definition the derivative
Differentiability	A curve that is smooth and contains no discontinuities or cusps; formally, a curve is differentiable at all values of the domain variable(s) for which the derivative exists
Fundamental Theorem of Calculus	The theorem that establishes the connection between derivatives, antiderivatives, and definite integrals Evaluation part of the FTC: If f is continuous on $[a,b]$, and F is any antiderivative of f, then $\int_a^b f(x)dx = F(b) - F(a)$ Antiderivative part of the FTC: If f is continuous on $[a,b]$, then $\frac{d}{dx}\int_a^x f(t)dt = f(x)$ for every x in $[a,b]$
Indefinite integral(s)	The family of functions that have a given function as a common derivative; the indefinite integral of $f(x)$ is written $\int f(x)dx$ [e.g., $\int x^2\, dx = \frac{1}{3}x^3 + C$]
Infinite limits	A limit that has an infinite result (either ∞ or – ∞), or a limit taken as the variable approaches ∞ (infinity) or – ∞ (negative infinity); the limit can be one-sided
Intermediate Value Theorem	If f is a function that is continuous over the domain $[a,b]$ and if m is a number between $f(a)$ and $f(b)$, then there is some number c between a and b such that $f(c) = m$
Limit(s)	The value that a function or expression approaches as the domain variable(s) approach a specific value; limits are written in the form $\lim_{x\to A} f(x)$ [e.g., the limit of $f(x) = \frac{1}{x}$ as x approaches 3 is $\frac{1}{3}$; this is written $\lim_{x\to 3}\frac{1}{x} = \frac{1}{3}$]

Mean Value Theorem	If function f is continuous on $[a, b]$ and differentiable on (a, b), then there exists a number c in (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$
Power Rule	A formula for finding the derivative of a power of a variable; $\frac{d}{dx}(x^n) = nx^{n-1}$
Product Rule	A formula for the derivative of the product of two functions; $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ or $(uv)' = u'v + uv'$
Quotient Rule	A formula for the derivative of the quotient of two functions; $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ or $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$
Tangent line	A line that touches a curve at a point without crossing over; formally, it is a line which intersects a differentiable curve at a point where the slope of the curve equals the slope of the line