Geometry

Curriculum Framework

2013

Compiled using the Common Core State Standards for Mathematics and the PARCC Model Content Frameworks for Mathematics Version 3.0 (November 2012 revision)

Course Title: Geometry
Course/Unit Credit: 1
Course Number: 431000
Teacher Licensure: Secondary Mathematics
Grades: 9-12
Prerequisite: Algebra I or Algebra A/B

Course Description: "The fundamental purpose of the course in Geometry is to formalize and extend students' geometric experiences from the middle grades. Students explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. Close attention should be paid to the introductory content for the Geometry conceptual category found in the high school CCSS. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations." - http://www.corestandards.org/

This document was created to delineate the standards for this course in a format familiar to the educators of Arkansas. For the state-provided Algebra A/B, Algebra I, Geometry A/B, Geometry, and Algebra II documents, the language and structure of the Common Core State Standards for Mathematics (CCSS-M) have been maintained. The following information is helpful to correctly read and understand this document.

"**Standards** define what students should understand and be able to do.

Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related." - http://www.corestandards.org/

"Standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B. . . . These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that standards are not just promises to our children, but promises we intend to keep." - http://www.corestandards.org/

This document includes only the standards designated as assessable by the PARCC Model Content Frameworks. The standards in this document appear exactly as written in the CCSS-M. Italicized portions of the standards offer clarification. The Plus Standards (+) from the Common Core State Standards for Mathematics may be incorporated into the curriculum to adequately prepare students for more rigorous courses (e.g., Advanced Placement, International Baccalaureate, or concurrent credit courses).

Geometry

Domain	Cluster	Course Emphases
Congruence		
	1. Experiment with transformations in the plane	Supporting
	2. Understand congruence in terms of rigid motions	Major
	3. Prove geometric theorems	Major
	4. Make geometric constructions	Supporting
Similarity, Right Triangles, and Trigonometry		
	5. Understand similarity in terms of similarity transformations	Major
	6. Prove theorems using similarity	Major
	7. Define trigonometric ratios and solve problems involving right triangles	Major
Circles		
	8. Understand and apply theorems about circles	Additional
	9. Find arc lengths and areas of sectors of circles	Additional
Expressing Geometric Properties with Equations		
	10. Translate between the geometric description and the equation of a conic section	Additional
	11. Use coordinates to prove simple geometric theorems algebraically	Major
Geometric measurement and dimension		
	12. Explain volume formulas and use them to solve problems	Additional
	13. Visualize relationships between two-dimensional and three-dimensional objects	Additional
Modeling with Geometry		
	14. Apply geometric concepts in modeling situations	Major

* *Asterisks identify potential opportunities to integrate content with the modeling practice*

Domain: Congruence
Cluster(s): 1. Experiment with transformations in the plane
2. Understand congruence in terms of rigid motions
3. Prove geometric theorems
4. Make geometric constructions

G.CO.1	1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	Supporting (CS, CIR)
G.CO.2	1	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	Supporting (CS)
G.CO.3	1	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	Supporting (CS)
G.CO.4	1	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	Supporting (CS)
G.CO.5	1	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	Supporting (CS)
G.CO.6	2	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	Major (CS)
G.CO.7	2	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	Major (CS)
G.CO.8	2	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	Major (CS)
G.CO.9	3	Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	Major (CS)
G.CO.10	3	Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	Major (CS)
G.CO.11	3	Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*	Major (CS)
G.CO.12	4	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*	Supporting (CS)
G.CO.13	4	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	Supporting (CS, CIR)

Key:

CCSS-M Domain and Standard #	CCSS-M Cluster	CCSS-M Standard	PARCC Course Emphases (Category)

Domain: Similarity, Right Triangles, and Trigonometry

Cluster(s): 5. Understand similarity in terms of similarity transformations
6. Prove theorems involving similarity
7. Define trigonometric ratios and solve problems involving right triangles

G.SRT.1	5	Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	Major (CS)
G.SRT.2	5	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	Major (CS)
G.SRT.3	5	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	Major (CS)
G.SRT.4	6	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.	Major (CS)
G.SRT.5	6	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	Major (CS)
G.SRT.6	7	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	Major (CS)
G.SRT.7	7	Explain and use the relationship between the sine and cosine of complementary angles.	Major (CS)
G.SRT.8	7	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*	Major (CS)

Key:

CCSS-M Domain and Standard #	CCSS-M Cluster	CCSS-M Standard	PARCC Course Emphases (Category)

Domain: Circles

Cluster(s): 8. Understand and apply theorems about circles
9. Find arc lengths and areas of sectors of circles

G.C.1	8	Prove that all circles are similar.	Additional (CIR)
G.C.2	8	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	Additional (CIR)
G.C.3	8	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	Additional (CIR)
G.C.5	9	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	Additional (CIR)

Domain: Expressing Geometric Properties with Equations

Cluster(s): 10. Translate between the geometric description and the equation of a conic section
11. Use coordinates to prove simple geometric theorems algebraically

G.GPE.1	10	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	Additional (CIR)
G.GPE.4	11	Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point* $(1, \sqrt{3})$ *lies on the circle centered at the origin and containing the point* $(0, 2)$.	Major (DG)
G.GPE.5	11	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	Major (CS)
G.GPE.6	11	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	Major (CS, DG)
G.GPE.7	11	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*	Major (CS, DG)

Key:

CCSS-M Domain and Standard #	CCSS-M Cluster	CCSS-M Standard	PARCC Course Emphases (Category)

Domain: Geometric measurement and dimension

Cluster(s): 12. Explain volume formulas and use them to solve problems

13. Visualize relationships between two-dimensional and three-dimensional objects

G.GMD.1	12	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	Additional (DG)
G.GMD.3	12	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*	Additional (DG)
G.GMD.4	13	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	Additional (DG)

Domain: Modeling with Geometry

Cluster(s): 14. Apply geometric concepts in modeling situations

G.MG.1	14	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*	Major (CS)
G.MG.2	14	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*	Major (DG)
G.MG.3	14	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*	Major (CS)

Common Core State Standards for Mathematical Practice

Make sense of problems and persevere in solving them Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
Reason abstractly and quantitatively Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
Construct viable arguments and critique the reasoning of others Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
Model with mathematics Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Use appropriate tools strategically

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Attend to precision

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Look for and make use of structure

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(x – y) 2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Look for and express regularity in repeated reasoning

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

The information contained in this document provides one way to frame discussions about scope and sequence of the standards leading to the development of local curriculum maps. This document incorporates the priorities assigned to each standard in the PARCC Model Content Frameworks (Major, Supporting, and Additional), which will help guide discussions related to priority, focus, and time allocated for each topic. In the spring of 2013, a committee composed of Arkansas educators categorized each standard into themes, which could frame the sequencing or organization of the content. These thematic categories are defined below. Districts are encouraged to approach the development of a curriculum in a manner that best serves the needs of their students.

Thematic categories for Geometry are Congruence and Similarity, Circle Geometry, and Dimensional Geometry.

- The theme, Congruence and Similarity, appears in each of the standards chosen for this topic.
  - Students will build on their previous knowledge of geometric terminology and relationships to compare figures.
- The theme, Circles, appears in each of the standards chosen for this topic.
  - Students will extend their previous knowledge and terminology to develop a deeper understanding of circles and their relationships to other geometric figures.
- The theme, Dimensional Geometry, appears in each of the standards chosen for this topic.
  - Students will apply and use geometric properties to solve equations algebraically and visualize relationships in both 2-dimensional and 3-dimensional shapes.

Frameworks Committee Categorization of Geometry Standards (indicated in the column to the far right of each standard):

- Congruence and Similarity (CS)
  - Standards include G.CO.1-13; G.SRT.1-8; G.GPE. 5-7; G.MG.1, 3
- Circles (CIR)
  - Standards include G.CO.1; G.C.1-3, 5; G.GPE. 1
- Dimensional Geometry (DG)
  - Standards include G.GPE. 4, 6, 7; G.GMD.1, 3, 4; G.MG.2