Pre-Calculus

Curriculum Framework

2012

Course Title: Pre-Calculus
Course/Unit Credit: 1
Course Number:
Teacher Licensure: Secondary Mathematics
Grades: 9-12

Pre-Calculus

Pre-Calculus will emphasize a study of trigonometric functions and identities as well as applications of right triangle trigonometry and circular functions. Students will use symbolic reasoning and analytical methods to represent mathematical situations, express generalizations, and study mathematical concepts and the relationships among them. Students will use functions and equations as tools for expressing generalizations. Teachers are responsible for including the eight Standards for Mathematical Practice found in the Common Core State Standards for Mathematics (CCSS-M). Pre-Calculus does not require Arkansas Department of Education approval.

Prerequisites: Algebra I, Geometry, Algebra II

Strand	Content Standard
Number and Quantity	
	1. Students will use complex numbers and determine how polar and rectangular coordinates are related.
	2. Students will perform operations with vectors and use those skills to solve problems.
Trigonometry	
	3. Students will develop and apply the definitions of the six trigonometric functions and use the definitions to solve problems and verify identities.
	4. Students will solve trigonometric equations and sketch the graph of periodic trigonometric functions.
Conic Sections	
	5. Students will identify, analyze, and sketch the graphs of the conic sections and relate their equations and graphs.
Functions	
	6. Students will be able to find the inverse of functions and use composition of functions to prove that two functions are inverses.
	7. Students will be able to interpret different types of functions and their key characteristics including polynomial, exponential, logarithmic, power, trigonometric, rational, and other types of functions.

Strand: Number and Quantity

Content Standard 1: Students will use complex numbers and determine how polar and rectangular coordinates are related.

		Connections to CCSS-M
NQ.1.PC.1	Find the conjugate of a complex number; use conjugates to find *moduli* and quotients of complex numbers	N.CN.3
NQ.1.PC.2	Represent complex numbers and their operations on the complex plane in rectangular and *polar form,* including real and imaginary numbers, and explain why the rectangular and *polar forms* of a given complex number represent the same number	N.CN.4
NQ.1.PC.3	Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation	N.CN.5
NQ.1.PC.4	Calculate the distance between numbers in the complex plane as the *modulus* of the difference; calculate the midpoint of a segment as the average of the numbers at its endpoints	N.CN.6

Key: NQ.1.PC.1 – Number and Quantity.Content Standard 1.Pre-Calculus.1st Student Learning Expectation

Strand: Number and Quantity

Content Standard 2: Students will perform operations with vectors and use those skills to solve problems.

		Connections to CCSS-M
NQ.2.PC.1	Recognize *vector* quantities as having both magnitude and direction; represent *vector* quantities by directed line segments and use appropriate symbols for *vectors* and their magnitudes (e.g., v, $\|v\|$, $\|\|v\|\|$, $\boldsymbol{v}$)	N.VM.1
NQ.2.PC.2	Solve problems involving velocity and other quantities that can be represented by *vectors*	N.VM.3
NQ.2.PC.3	Perform operations on *vectors* in component form: • identify *vector* components from an initial and terminal point • *scalar* multiplication • *vector* addition and subtraction	N.VM.4, N.VM.5
NQ.2.PC.4	Represent and perform *vector* operations geometrically • *scalar* multiplication • *vector* addition (triangle and parallelogram models) • *vector* subtraction (adding a negative *vector*, missing addend model)	N.VM.4, N.VM.5
NQ.2.PC.5	Compute the magnitude of a *scalar* multiple $c\boldsymbol{v}$ using $\|\|cv\|\| = \|c\|v$; compute the direction of cv knowing that when $\|c\|v \neq 0$, the direction of cv is either along $v\ (for\ c > 0)$ or against $v\ (for\ c < 0)$	N.VM.5b

Key: NQ.2.PC.1 – Number and Quantity.Content Standard 2.Pre-Calculus.1st Student Learning Expectation

Strand: Trigonometry

Content Standard 3: Students will develop and apply the definitions of the six trigonometric functions and use the definitions to solve problems and verify identities.

		Connections to CCSS-M
T.3.PC.1	Use special triangles to determine geometrically the values of sine, cosine, and tangent for $\pi/3$, $\pi/4$ and $\pi/6$; use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number	F.TF.3
T.3.PC.2	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems	F.TF.9
T.3.PC.3	Derive the formula $A = \left(\frac{1}{2}\right) ab \sin C$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side	G.SRT.9
T.3.PC.4	Prove the *Law of Sines* and the *Law of Cosines* and use them to solve problems	G.SRT.10
T.3.PC.5	Understand and apply the *Law of Sines* and the *Law of Cosines* to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces)	G.SRT.11
T.3.PC.6	Define and use reciprocal functions, cosecant, secant, and cotangent to solve problems	Not Applicable

Key: T.3.PC.1 – Trigonometry.Content Standard 3.Pre-Calculus.1st Student Learning Expectation

Strand: Trigonometry

Content Standard 4: Students will solve trigonometric equations and sketch the graph of periodic trigonometric functions.

		Connections to CCSS-M
T.4.PC.1	Use the unit circle to explain symmetry (odd and even) and periodicity of *trigonometric functions*	F.TF.4
T.4.PC.2	Understand that restricting a *trigonometric function* to a domain on which it is always increasing or always decreasing allows its inverse to be constructed	F.TF.6
T.4.PC.3	Use *inverse functions* to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology and interpret the solutions in terms of the context	F.TF.7
T.4.PC.4	Recognize that some trigonometric equations have infinitely many solutions and be able to state a general formula to represent the infinite solutions	Not Applicable
T.4.PC.5	Use *trigonometric functions* to model physical situations (e.g., harmonic motion, circular motion, area of polygons)	F.TF.5

Key: T.4.PC.1 – Trigonometry.Content Standard 4.Pre-Calculus.1st Student Learning Expectation

Strand: Conic Sections

Content Standard 5: Students will identify, analyze, and sketch the graphs of the conic sections and relate their equations and graphs.

		Connections to CCSS-M
CS.5.PC.1	Derive the equations of ellipses and hyperbolas given the foci using the fact that the sum or difference of distances from the foci is constant	G.GPE.3
CS.5.PC.2	Find the equations for the *asymptotes* of a hyperbola	Not Applicable
CS.5.PC.3	Complete the square in order to generate an equivalent form of an equation for a conic section; use that equivalent form to identify key characteristics of the conic section	Not Applicable
CS.5.PC.4	Identify, graph, write, and analyze equations of each type of conic section, using properties such as symmetry, intercepts, foci, *asymptotes*, and *eccentricity*, and using technology when appropriate	Not Applicable
CS.5.PC.5	Solve systems of equations and inequalities involving conics and other types of equations, with and without appropriate technology	Not Applicable

Key: CS.5.PC.1 – Conic Sections.Content Standard 5.Pre-Calculus.1st Student Learning Expectation

Strand: Functions

Content Standard 6: Students will be able to find the inverse of functions and use composition of functions to prove that two functions are inverses.

		Connections to CCSS-M
F.6.PC.1	Compose functions [e.g., if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time]	F.BF.1c
F.6.PC.2	Verify by composition that one function is the inverse of another	F.BF.4b
F.6.PC.3	Read values of an *inverse function* from a graph or a table given that the function has an inverse	F.BF.4c
F.6.PC.4	Produce an invertible function from a non-invertible function by restricting the domain	F.BF.4d
F.6.PC.5	Combine standard function types using arithmetic operations (e.g., build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model)	F.BF.1b
F.6.PC.6	Understand the inverse relationship between exponents and logarithms; use this relationship to solve problems involving logarithms and exponents	F.BF.5

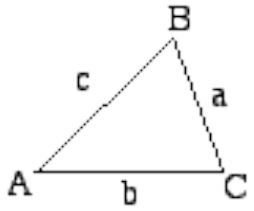
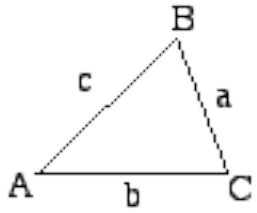
Key: F.6.PC.1 – Functions.Content Standard 6.Pre-Calculus.1st Student Learning Expectation

Strand: Functions

Content Standard 7: Students will be able to interpret different types of functions and their key characteristics including polynomial, exponential, logarithmic, power, trigonometric, rational, and other types of functions.

		Connections to CCSS-M
F.7.PC.1	Graph rational functions identifying zeros and *asymptotes* when suitable factorizations are available and show end behavior	F.IF.7d
F.7.PC.2	Analyze and interpret power and polynomial functions numerically, graphically, and algebraically, identifying key characteristics such as intercepts, end behavior, domain and range, relative and absolute maximum and minimum, as well as intervals over which the function increases and decreases	Not Applicable
F.7.PC.3	Analyze and interpret rational functions numerically, graphically, and algebraically, identifying key characteristics such as *asymptotes* (vertical, horizontal, and slant), domain and range, end behavior, *point discontinuities*, and intercepts	Not Applicable
F.7.PC.4	Analyze and interpret *exponential functions* numerically, graphically, and algebraically, identifying key characteristics such as *asymptotes*, domain and range, end behavior, and intercepts	Not Applicable
F.7.PC.5	Analyze and interpret *logarithmic functions* numerically, graphically, and algebraically, identifying key characteristics such as *asymptotes*, domain and range, end behavior, and intercepts	Not Applicable
F.7.PC.6	Analyze and interpret *trigonometric functions* numerically, graphically, and algebraically, identifying key characteristics such as period, midline, domain and range, *amplitude*, *phase shift*, and *asymptotes*	Not Applicable
F.7.PC.7	Build functions to model real-world applications using algebraic operations on functions and composition, with and without appropriate technology (e.g., profit functions as well as volume and surface area, optimization subject to constraints)	Not Applicable

Key: F.7.PC.1 – Functions.Content Standard 7.Pre-Calculus.1st Student Learning Expectation

Amplitude	Half the difference between the minimum and maximum values of the range; only periodic functions with a bounded range have an amplitude
Asymptote(s)	Line(s) to which a graph becomes arbitrarily close as the value of x or y increases or decreases without bound (e.g., vertical, horizontal, slant)
Eccentricity	A number that indicates how drawn out or attenuated a conic section is; eccentricity is represented by the letter e (no relation to $\mathrm{e} = 2.718..$)
Exponential Function(s)	Function(s) in which the variable(s) occurs in the exponent [e.g., $f(x) = ab^x, b > 0$]
Inverse Function(s)	Two functions f and g are inverse functions, if and only if both their compositions yield the identity function {e.g., $[f \circ g](x) = x$ and $[g \circ f](x) = x$}
Law of Cosines	An equation relating the cosine of an interior angle and the lengths of the sides of a triangle; the Pythagorean theorem is a corollary of the Law of Cosines $c^2 = a^2 + b^2 - 2ab\cos C$ $b^2 = a^2 + c^2 - 2ac\cos B$ $a^2 = b^2 + c^2 - 2bc\cos A$ 
Law of Sines	Equations relating the sines of the interior angles of a triangle and the corresponding opposite sides $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ 
Logarithmic Functions	Function of the form $y = \log_b x$, where $b > 0, x > 0$ and $b \neq 1$
Modulus (pl. Moduli)	For complex number(s) in polar form $z = \mathrm{r}(\cos\theta + i\sin\theta)$ the modulus is r
Phase Shift	Horizontal shift for a periodic function
Point Discontinuities	A point at which the graph of a relation or function is not connected

Polar Form(s) of a Complex Number	The polar form(s) or trigonometric form(s) of the complex number $z = a + bi$ is $z = \mathrm{r}(\cos\theta + i\sin\theta)$ where $a = r\cos\theta$, $b = r\sin\theta$, $r = \sqrt{a^2 + b^2}$, and $\tan\theta = b/a$
Scalar	Any real number, or any quantity that can be measured using a single real number; temperature, length, and mass are all scalars; a scalar is said to have magnitude but no direction
Trigonometric Function(s)	The six functions are sine, cosine, tangent, cosecant, secant, and cotangent
Vector(s)	Quantity or quantities with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers